

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 7

- All the problems this week are similar to the ones in the assessed coursework.
- The instructor will present only one starred problem, leaving you more time to work on and get help with the other problems.
- Since there is not time to do all the problems in the tutorial, it is important to study the solutions once they become available online at 3pm on Friday.
- Although tutorial problems are unassessed, they are in the same style (apart from the starred questions) as the problems in the final exam, so you are advised to study all of the tutorial problems and solutions very carefully.

(*)Problem 1: Find the domain, range, domain boundary and level curves for each of the functions given below. Also determine if the domain is open or closed or neither, and if it is bounded or unbounded.

$$(a) \quad f(x, y) = \frac{1}{\sqrt{16 - x^2 - y^2}}; \quad (b) \quad f(x, y) = \ln(x^2 + y^2); \quad (c) \quad \sin^{-1}(y - x).$$

Problem 2: Find the equation of the level curve through the point $(2\sqrt{2}, \sqrt{2})$ for the function $f(x, y) = 16 - x^2 - y^2$ and the equation of the level surface through the point $(3, -1, 1)$ for the function $f(x, y, z) = \sqrt{x - y} - \ln z$.

Problem 3: Find the following limits:

$$(a) \quad \lim_{(x,y) \rightarrow (1,1), x \neq y} \frac{x^2 - 2xy + y^2}{x - y}; \quad (b) \quad \lim_{(x,y) \rightarrow (2,0), 2x-y \neq 4} \frac{\sqrt{2x-y} - 2}{2x - y - 4}.$$

Problem 4: Find all the points (x, y) or (x, y, z) at which the following functions are continuous:

$$(a) \quad f(x, y) = \sin(x + y); \quad (b) \quad f(x, y) = \sin \frac{1}{xy}; \quad (c) \quad f(x, y) = xy \sin \frac{1}{z}.$$

Problem 5: By considering different paths of approach, show that the following functions have no limit as $(x, y) \rightarrow (0, 0)$:

$$(a) \quad f(x, y) = \frac{x^4 - y^2}{x^4 + y^2}; \quad (b) \quad f(x, y) = \frac{x - y}{x + y}.$$

Problem 6: By using polar coordinates $(x = r \cos \theta, y = r \sin \theta)$, either find the limit of the following functions as $(x, y) \rightarrow (0, 0)$ or show that the limit does not exist:

$$(a) \quad f(x, y) = \frac{x^3 - xy^2}{x^2 + y^2}; \quad (b) \quad f(x, y) = \frac{y^2}{x^2 + y^2}; \quad (c) \quad f(x, y) = \tan^{-1} \left(\frac{|x| + |y|}{x^2 + y^2} \right).$$

Problem 7: Find all the second-order partial derivatives for the following functions:

$$(a) \quad f(x, y) = x + y + xy; \quad (b) \quad f(x, y) = x^2y + \cos y + y \sin x.$$

Problem 8: Find the values of $\partial z / \partial x$ and $\partial z / \partial y$ at the point $(1, 1, 1)$ of the curve $z(x, y)$ defined implicitly by the equation $xy + z^3x - 2yz = 0$.